EXPECTED ACTIVATION LEVEL OF KNOWLEDGE ABOUT FUNCTIONS: A COMPARISON CHILE/FRANCE/ITALY

Charlotte Derouet*, Carolina Henriquez**, Romina Menares**, Monica Panero***

* LDAR, Université de Paris Diderot ; ** Pontificia Universidad Catolica de Valparaiso ; *** Dipartimento di Matematica, Università degli Studi di Torino

Keywords: function, international comparison, transition high school/university.

CONTEXT OF THE STUDY

The aim of the poster is to have insight of what is expected from students at the end of secondary school. This can be a way to study the prerequisites "at the entrance" to university courses in mathematics. Analysing the tasks proposed in the high school final assessment seems to be a first possible approach. In this study, we want to present a comparison between three countries: Chile, France and Italy. The comparison concerns the work on functions at the end of the secondary school. First of all, these three countries have a different school system. The Chilean high school finishes at grade 12. The final assessment, called "Prueba de Selección Universitaria" (PSU), is an exam that ranks students for accessing to university. However the whole set of the notions studied at secondary school is not evaluated though PSU. The PSU test in mathematics is a multiple-choice test. In France, the high school goes from grade 10 to grade 12. At the end of this period, students have to pass an exam, called "Baccalauréat", which is compulsory to enter university. The "Baccalauréat" varies according to the sections. In scientific section, the exam in mathematics is composed of four exercises, with detailed questions. Finally, the Italian high school finishes at grade 13 (so it lasts one year more than the Chilean one and the French one). The final exam is called "Maturità" and, as in France, it is necessary to access to university. In scientific section, the exam in mathematics consists of two problems, of which only one has to be solved, and 10 questions (the candidate chooses and solves 5 of them). Clearly, preparing students for the final assessment represents one of the main aims of the last year of high school in each country. In our study, we focus on the tasks involving functions. And we wonder: what activation level of knowledge related to functions is expected from students at the end of the secondary school in Chile, France and Italy? To answer this research question, we propose an international comparison through the analysis of the different final assessments.

A PRIORI ANALYSIS OF TASKS

In the poster we show one representative task on functions for each of the three countries and, through an a priori analysis, we try to detect particularities in each resolution process. We partially refer to the methodology of tasks analysis introduced by Aline Robert (1998). Specifically, we wonder if the question is open or closed, we focus on the activated frames (Douady, 1986), working frames and registers (Duval, 1995). Moreover, we consider the adaptations to do (introducing steps, choosing a method, recognising the modality of application) as well as the expected activation level of knowledge (technical, mobilisable, available). In particular, we can distinguish two degrees of availability. On the one hand, a certain notion/property can be recalled and employed as an "object": for example, the memorisation of a formula to directly work the involved notion. We call it "object availability". On the other hand, a notion/property can be recalled and introduced by the student himself as a "tool", to solve a question that doesn't involve directly the notion. We call it "tool availability". The degree of availability of knowledge is at the core of our comparison.

CONCLUSIONS

This analysis allows us to notice some similarities and several remarkable differences between the countries assessments. Our main result is the observation of a great dissimilarity at the level of availability and of autonomy expected from the students. Chilean students are required to have a high activation level of object available knowledge. French students are expected to mobilise some pieces of knowledge at a tool available level, but the object available knowledge prevails, with little space left to autonomy. Italian students are supposed to be more autonomous in solving tasks and in mobilising knowledge at a high level of availability as a tool.

REFERENCES

Douady, R. (1986). Jeux de cadres et dialectique outil/objet, *Recherches en didactique des mathématiques*, vol. 7, n°2, pp. 5-32.

Duval, R. (1995). *Sémiosis et pensée humaine. Registres sémiotiques et apprentissages intellectuels*, Peter Lang, Berne.

Robert, A. (1998). Outils d'analyse des contenus mathématiques à enseigner au lycée et à l'Université. *Recherches en didactique des mathématiques*, vol. 18, n° 2, pp. 139-190.

International comparison between final secondary assessments

Charlotte Derouet*, Carolina Henríquez**, Romina Menares**, **Monica Panero*****
* Laboratoire de Didactique André Revuz, Université Paris Diderot ; ** Pontificia Universidad Católica de Valparaíso ; *** Dipartimento di Matematica, Università degli Studi di Torino
monica.panero@unito.it

Project ECOS-CONICYT C13H03.

Context of the study and research question

Three countries : Chile, France and Italy.
Aim : What is expected from students at the end of secondary school.
Focus : Study of function (functional frame).
Research question : What activity is expected from students ?

Methodology : *a priori* analysis of one representative assessment task for each country.

1. Grade of opening
2. Working frames
3. Registers
4. Adaptations to do
5. Level of activation of knowledge

A priori analysis of some assessment tasks

Chile

Prueba de Selección Universitaria (PSU) – Grade 12

- Ranks students for accessing to university
- Multiple-choice test

¿Cuál de las siguientes afirmaciones es **FALSA**, con respecto a la función

$f(x) = -(x^2 - 4)$, cuando x recorre todos los números reales?

A) La función toma un valor máximo.
B) Las ramas de la parábola asociada a la función se abren hacia abajo.
C) La gráfica de la función intersecta al eje de las ordenadas en el punto (0, -4).
D) La gráfica de la función intersecta al eje de las abscisas en los puntos (2,0) y (-2,0).
E) El eje de simetría de la gráfica de la función es el eje y.

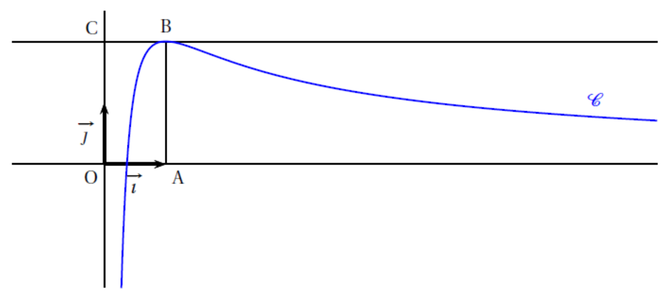
1. Semi-closed question (***autonomy in the choice of methods***)
2. Algebraic / Numerical / Graphical (*not all indicated ; no frame changes*)
3. In the question : Natural language / Algebraic expression
   In the possible resolutions : Algebraic expression / Numerical
   expression / Graphical representation (*some indications for choosing and changing register* ***; not to be made explicit***)
4. **Point of view changes**, method choices
5. **Availability as object** (*recalling and employing properties of secondary degree functions*) No justification required

France

Baccalauréat – Grade 12 Scientific section

- Exam, compulsory to enter university
- Exercises

Sur le graphique ci-dessous, on a tracé, dans le plan muni d'un repère orthonormé $(O; \vec{\imath}, \vec{\jmath})$, la courbe représentative $\mathscr{C}$ d'une fonction f définie et dérivable sur l'intervalle $]0\,;+\infty[$.



On dispose des informations suivantes :
— les points A, B, C ont pour coordonnées respectives (1 , 0), (1 , 2), (0 , 2) ;
— la courbe $\mathscr{C}$ passe par le point B et la droite (BC) est tangente à $\mathscr{C}$ en B ;
— il existe deux réels positifs a et b tels que pour tout réel strictement positif x,

$$f(x) = \frac{a + b\ln x}{x}.$$

2. a. Justifier que pour tout réel x appartenant à l'intervalle $]0\,,+\infty[$, $f'(x)$ a le même signe que $-\ln x$.
b. Déterminer les limites de f en 0 et en $+\infty$. On pourra remarquer que pour tout réel x strictement positif, $f(x) = \frac{2}{x} + 2\frac{\ln x}{x}$.
c. En déduire le tableau de variations de la fonction f.

1. Semi-closed question (***questions enchainment, autonomy in the choice of methods, some helps***)
2. Algebraic / "Algebraised" analysis / Functional (*indicated frame changes*)
3. In the question : Natural language / Algebraic expression
   In the possible resolution : Natural language / Algebraic and "Algebraised" analysis expressions / Table (*indicated register changes*)
4. **Recognising modality of application, method choices, use of previous questions, point of view changes**
5. **Availability as object** (*application of theorems*)

Italy

Maturità – Grade 13 Scientific section

- Exam, compulsory to enter university
- Problems and questions, free choice

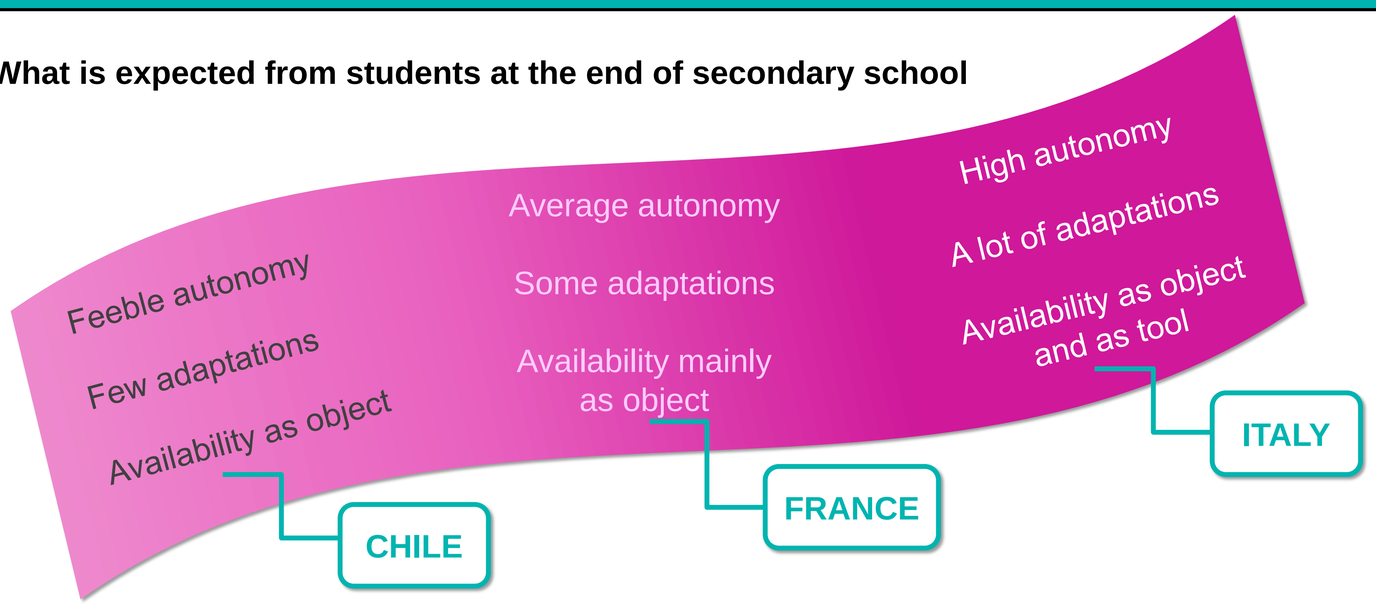
Sia f la funzione definita per tutti gli x positivi da $f(x) = x^3 \ln x$.

1. Si studi f e si tracci il suo grafico γ su un piano riferito ad un sistema di assi cartesiani ortogonali e monometrici Oxy; accertato che γ presenta sia un punto di flesso che un punto di minimo se ne calcolino, con l'aiuto di una calcolatrice, le ascisse arrotondate alla terza cifra decimale.

1. Semi-open question (***open except for one anticipation on the results***)
2. Functional / Algebraic / "Algebraised" analysis / Numerical → Graphical (*not all indicated ; autonomous frame changes*)
3. In the question : Natural language / Algebraic expression
   In the possible resolution : Algebraic and "Algebraised" analysis expressions / Table / Graphical
   representation / Natural language (*few indications for* ***choosing and changing registers***)
4. **Introduction of steps, point of view changes, anticipation, reasoning by contradiction, …**
5. **Availability as object** (*recalling and employing properties of functions*) **Availability as tool** (*recalling and introducing rules to help the reasoning process*)

Conclusions

What is expected from students at the end of secondary school



References

DEROUET, C. & PANERO, M. (2014). *Étude comparative sur l'enseignement des fonctions dans le secondaire en France et en Italie*, Cahiers du Laboratoire de Didactique André Revuz, n°11.

DOUADY, R. (1986), Jeux de cadres et dialectique outil/objet, *Recherches en didactique des mathématiques* 7(2), 5-32.

DUVAL, R. (1995), Sémiosis et pensée humaine, *Peter Lang, Bern.*

ROBERT, A. (1998), Outils d'analyse des contenus mathématiques à enseigner au lycée et à l'Université. *Recherches en didactique des mathématiques*, vol. 18, n° 2, pp. 139-190.